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ScienceDirect
Applied Energy 84 (2007) 882–892
APPLIED ENERGY
www.elsevier.com/locate/apenergy

Performance analysis and parametric optimal design of an irreversible multi-couple thermoelectric refrigerator under various operating conditions

Yuzhuo Pan [a,b], Bihong Lin [a,b], Jincan Chen [a,*]

[a] *Department of Physics, Xiamen University, Xiamen 361005, People's Republic of China*
[b] *Department of Physics, Quanzhou Normal University, Quanzhou 362000, People's Republic of China*

Received 23 January 2007; accepted 3 February 2007
Available online 16 May 2007

Abstract

The performance of a thermoelectric refrigeration device, consisting of multi-couple thermoelectric elements and operating between two heat-reservoirs at constant temperatures, is investigated. The influence of the external and internal irreversibilities of the thermoelectric refrigeration device on the performance of the system is analyzed. The general expressions of the coefficient of performance and power input are derived by introducing some dimensionless parameters and variables. The coefficient of performance of the refrigeration device is maximized for a given cooling-load and total heat-transfer area of the system, and consequently, the structure parameters of the device and the area ratio of the heat exchangers of the system are determined optimally. The effects of the various parameters on the optimal performance of the device are discussed further. The results obtained here will be useful for a more detailed investigation and for the optimal design and manufacture of real thermoelectric refrigeration devices.

Keywords: Thermoelectric refrigeration device; Irreversibility; Structure parameters; Heat-transfer area; Performance analysis; Optimal

* Corresponding author. Tel.: +86 592 2189 426; fax: +86 592 2185 119.
E-mail address: jcchen@xmu.edu.cn (J. Chen).

0306-2619/$ - see front matter
doi:10.1016/j.apenergy.2007.02.008

1. Introduction

Thermoelectric refrigeration and generation devices occupy a niche market, because they are quiet and reliable, and friendly to our environment [1–4]. Nowadays, the devices have a distinct place in medical applications, scientific equipment and other applications, where a high-precision temperature-control is essential. However, the coefficients of performance of refrigeration devices and the efficiency of generation devices are low compared with traditional devices [5,6]. Therefore, the improvement of performance of the thermoelectric devices is an important issue in the future applications not only for developing high technology but also for improving our quality of life.

Generally, the coefficient of performance of thermoelectric refrigeration-devices or the efficiency of thermoelectric-generation devices is mainly determined by the parameter $Z = \alpha^2/(RK)$ which is called the figure of merit, where α is the Seebeck coefficient, R is the electric resistance, and K is thermal conductance [7,8]. It is well known that the larger the figure of merit, the better the performance of a thermoelectric device and the higher the efficiency of a thermoelectric generator [1,3,9–12]. In the case of given semiconductor materials, the figure of merit Z depends on the structure of the device. It has been proven that when $l_n/S_n = (l_p/S_p)\sqrt{\kappa_n\rho_p/(\kappa_p\rho_n)}$, Z attains its maximum and the geometric configuration of the thermoelectric device is optimum [13], where l and S are, respectively, the length and cross-sectional area of the semiconductor elements, ρ is the electrical resistivity, κ is the thermal conductivity, and the subscripts n and p designate the n- and p-type elements. On the other hand, for real thermoelectric devices, the operative conditions are different from each other and the thermal conductances between the thermoelectric device and the external heat reservoirs are always finite. These questions are necessarily considered in the optimal design of thermoelectric devices. This has been noted in recent years [14–17].

In the present paper, we will investigate the optimal performance of a thermoelectric refrigeration device with external and internal irreversibilities and analyze how the different operative conditions, the finite-rate heat transfer between the thermoelectric device and the external heat reservoirs, the structure parameters of the semiconductor elements and the area ratio of heat exchangers affect the performance of a multi-couple thermoelectric refrigeration device. The optimally operating regions of some important performance parameters of the thermoelectric refrigerator are determined.

2. An irreversible thermoelectric refrigeration device

In thermoelectric materials, heat can be transported or used to generate electricity based on the Peltier and Seebeck effects. According to non-equilibrium thermodynamics [18,19], when an electric current with the density $\mathbf{J}$ traverses a semiconductor placed in a temperature gradient, one has

$$\nabla \cdot \mathbf{J_E} = -\nabla \cdot (\kappa \nabla T) + T\mathbf{J} \cdot \nabla \alpha - \mathbf{J} \cdot \mathbf{J}/\sigma \qquad (1)$$

where $\mathbf{J_E}$ is the current density of energy in the semiconductor, T is the temperature inside the semiconductor, and α, κ, and σ are, respectively, the Seebeck coefficient, thermal conductivity, and electrical conductivity of the semiconductor.

A multi-couple thermoelectric refrigeration system, as shown in Fig. 1, consists of a semiconductor thermoelectric device and two heat exchangers at the hot and cold sides.

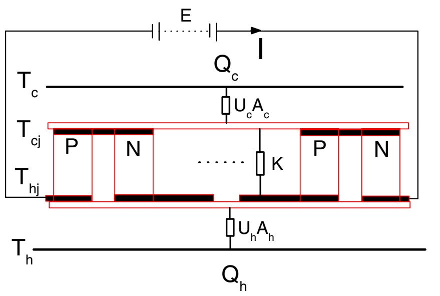

Fig. 1. A schematic diagram of a multi-couple thermoelectric refrigeration device.

The semiconductor thermoelectric device is composed of many n- and p-type semiconductor legs that are connected electrically in series by metal strips and thermally in parallel. The thermoelectric refrigerating-elements are assumed to be insulated, both electrically and thermally, from their surroundings, except at the junction-reservoir contacts. The current is assumed to flow in one dimension, as along the arm of the refrigeration device. In addition, it is assumed that the Seebeck coefficients α_n and α_p, the electrical resistivities ρ_n and ρ_p, and the thermal conductivities κ_n and κ_p of the n- and p-type semiconductor materials are independent of temperature in order to obtain some analytical solutions. When the thermoelectric refrigeration device operates stably, using Eq. (1) and the boundary condition $T_1(0) = T_2(0) = \cdots = T_i\ (0) = \cdots T_n(0) = T_{hj}$ and $T_1(l_p) = T_2(l_n) = \cdots = T_i\ (l_n) = \cdots T_n(l_n) = T_{cj}$, we can derive equations of heat conduction of a multi-couple thermoelectric refrigeration device as follows:

$$Q_c = \alpha I T_{cj} - \frac{1}{2} I^2 R - K(T_{hj} - T_{cj}) \tag{2}$$

and

$$Q_h = \alpha I T_{hj} + \frac{1}{2} I^2 R - K(T_{hj} - T_{cj}) \tag{3}$$

where Q_h and Q_c are the heat flows from the device to the heat sink and from the cooled space to the device, $\alpha = (\alpha_p - \alpha_n)n$, $R = (\rho_p l_p/S_p + \rho_n l_n/S_n)n$ and $K = (\kappa_p S_p/l_p + \kappa_n S_n/l_n)n$ are, respectively, the Seebeck coefficient, electrical resistance and thermal conductance of the semiconductor couples, n is the total number of couples, and I is the working electrical current.

According to the important relation between the structure parameters l_n/S_n and l_p/S_p mentioned above, the electrical resistance and thermal conductance of a multi-couple thermoelectric refrigeration-device can be, respectively, expressed as

$$R = n\left(\rho_p + \sqrt{\rho_p \rho_n \kappa_n / \kappa_p}\right)(l_p/S_p) \tag{4}$$

and

$$K = n\left(\kappa_p + \sqrt{\kappa_p \kappa_n \rho_n / \rho_p}\right)(S_p/l_p) \tag{5}$$

It can be found from Eqs. (2) and (3) that the internal irreversibility comes from Joule's heat I^2R due to the electrical current and heat leak $K(T_{hj} - T_{cj})$ due to the temperature difference between the hot and the cold junctions of the device. Due to the finite-rate heat

transfer between the thermoelectric refrigeration-device and the heat reservoir, the temperatures T_{hj} and T_{cj} of the thermoelectric refrigeration device are different from those of the heat reservoirs, T_h and T_c, and $T_{hj} > T_h > T_c > T_{cj}$. Assuming that the heat transfer between the hot and cold junctions of the thermoelectric refrigerator and their respective reservoirs obeys Newton's law [18,20], one has

$$Q_c = U_c A_c (T_c - T_{cj}) \tag{6}$$

and

$$Q_h = U_h A_h (T_{hj} - T_h) \tag{7}$$

where U_h and U_c are the overall heat-transfer coefficients of the heat exchangers at the hot and cold sides, and A_h and A_c are the heat-transfer areas of the hot and cold junctions.

By using Eqs. (2), (3), (6), and (7), T_{hj} and T_{cj} can be, respectively, expressed as

$$T_{hj} = \frac{K(U_cA_cT_c + U_hA_hT_h) + U_cA_cU_hA_hT_h + U_hA_hT_h\alpha I + (K + U_cA_c/2)RI^2 + R\alpha I^3/2}{K(U_cA_c + U_hA_h) + U_cA_cU_hA_h - (\alpha I)^2 + (U_hA_h - U_cA_c)\alpha I} \tag{8}$$

and

$$T_{cj} = \frac{K(U_cA_cT_c - U_hA_hT_h) + U_hA_hU_cA_cT_c - U_cA_cT_c\alpha I + (K + U_hA_h/2)RI^2 - R\alpha I^3/2}{K(U_cA_c + U_hA_h) + U_cA_cU_hA_h - (\alpha I)^2 + (U_hA_h - U_cA_c)\alpha I}. \tag{9}$$

Substituting from Eqs. (8) and (9) into Eqs. (6) and (7), one can obtain

$$q_c = \frac{(\theta_h - 1)(1 - x)x - j(x + C_1 j)(1 - x) - \frac{1}{2}j^2[1/(ZT_h)](x + C_1 j + 2C_1)(1 - x)}{[(1 - x) + C_2 - C_2 j](x + C_1 j) - [(1 - x) - C_2 j]C_1} \tag{10}$$

and

$$q_h = \frac{(1 - \theta_c)(1 - x)x - j[(1 - x) - C_2 j]x + \frac{1}{2}j^2[1/(ZT_h)][(1 - x) - C_2 j + 2C_2]x}{[(1 - x) + C_2 - C_2 j](x + C_1 j) - [(1 - x) - C_2 j]C_1} \tag{11}$$

where $q_h = Q_h/(KT_c)$, $q_c = Q_c/(KT_c)$, $\theta_h = 1/\theta_c = T_h/T_c$, $j = \alpha I/K$, $Z = \alpha^2/(RK)$, $C_1 = K/(U_h A)$, $C_2 = K/(U_c A)$, and $x = A_h/A$ are the dimensionless quantities, while $A = A_h + A_c$ is the total heat-transfer area of the heat exchangers.

Using Eqs. (10) and (11), one can derive the coefficient of performance and power input of the thermoelectric refrigeration system. They can be, respectively, expressed as

$$\varepsilon = \frac{(x-1)x[2ZT_h(\theta_h - j - 1) + j^2\theta_h] + C_1 j^2[2(x - ZT_h) - (j-2)(x-1)\theta_h]}{C_1 j^2(x-1)[j\theta_h - 2(ZT_h + \theta_h)] + xj^2\theta_h[2 + C_2(2+j) - 2x] + 2xjZT_h(x - 1 + \theta_h + C_2 j\theta_h - x\theta_h)} \tag{12}$$

and

$$p = \frac{(\theta_h + \theta_c)(x-1)x + j^2(C_1 - C_1 x + C_2 x) + \frac{1}{2}j^2[1/(ZT_h)][2x(1 - x - C_2 + C_1) - xj(C_1 + C_2) + C_1(j+2)]}{[(1-x) + C_2 - C_2 j](x + C_1 j) - [(1-x) - C_2 j]C_1} \tag{13}$$

where $p = P/(KT_c) = q_h - q_c$.

3. Performance analysis and parametric optimum design

For given semiconductor materials and specified operation conditions, the parameters ZT_h, U_hA, U_cA and θ_h are some known quantities. It is seen from Eqs. (10) and (12) that, if Q_c/T_c is given, the coefficient of performance is only a function of both K and x. Using Eqs. (10) and (12), one can plot the three-dimensional graph of the coefficient of performance varying with the parameters K and x, as shown in Fig. 2, where the parameters $Q_c/T_c = 0.003\ \mathrm{W\,K^{-1}}$, $\theta_h = 1.2$, $ZT_h = 1$ and $U_cA = U_hA = UA = 5\ \mathrm{W\,K^{-1}}$ are chosen. It is seen, from Fig. 2, that the coefficient of performance of the refrigeration system first increases and then decreases as K and x are increased. It shows that there exist the maximum coefficient of performance ε_{max} and the corresponding optimal values of K and x for other given parameters. It can be seen from Eqs. (10) and (12) and Fig. 2 that for the different choices of Q_c/T_c, K and x will have different optimal values. Using the extremal conditions $\partial\varepsilon/\partial K = 0$ and $\partial\varepsilon/\partial x = 0$ and Eqs. (10) and (12), one can generate the $\varepsilon_{max} \sim Q_c/T_c$, $j_{opt} \sim Q_c/T_c$, $K_{opt} \sim Q_c/T_c$, $x_{opt} \sim Q_c/T_c$, and $\varepsilon_{max} \sim K_{opt}$ characteristic curves of the multi-couple thermoelectric refrigeration system, as shown in Figs. 3–6, respectively, where the parameters $\theta_h = 1.2$ and $ZT_h = 1$ are chosen and curves I and II correspond to the cases of $U_cA = U_h$, $A = 10\ \mathrm{W\,K^{-1}}$ and $5\ \mathrm{W\,K^{-1}}$, respectively. It is seen from these curves that the maximum coefficient of performance ε_{max} is a monotonically decreasing function of Q_c/T_c, while j_{opt}, K_{opt} and x_{opt} are monotonically increasing functions of Q_c/T_c.

It is seen from Eq. (5) and Fig. 4 that, for a given Q_c/T_c, the optimal values of the parameters $n(S_p/l_p)$ and x can be obtained from these optimal-relation curves. For example, when $\theta_h = 1.2$, $ZT_h = 1$, $U_cA = U_hA = 5\ \mathrm{W\,K^{-1}}$ and $Q_c/T_c = 0.003\ \mathrm{W\,K^{-1}}$, the maximum coefficient of performance occurs at $K = K_{opt} = 0.0255\ \mathrm{W\,K^{-1}}$ and $x = x_{opt} = 0.62$. Then, the optimum value of $n(S_p/l_p)$ is

$$\left(n\frac{S_p}{l_p}\right)_{opt} = \frac{0.0255}{\kappa_p + \sqrt{\kappa_p\kappa_n\rho_n/\rho_p}} \tag{14}$$

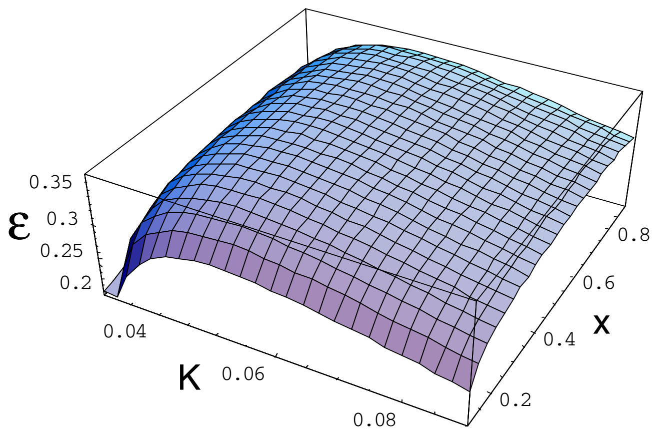

Fig. 2. The variation of the coefficient of performance ε with the thermal conductance K and area ratio x, where the parameters $Q_c/T_c = 0.003\ \mathrm{W\,K^{-1}}$, $\theta_h = 1.2$, $ZT_h = 1$ and $U_cA = U_hA = UA = 5\ \mathrm{W\,K^{-1}}$ are chosen.

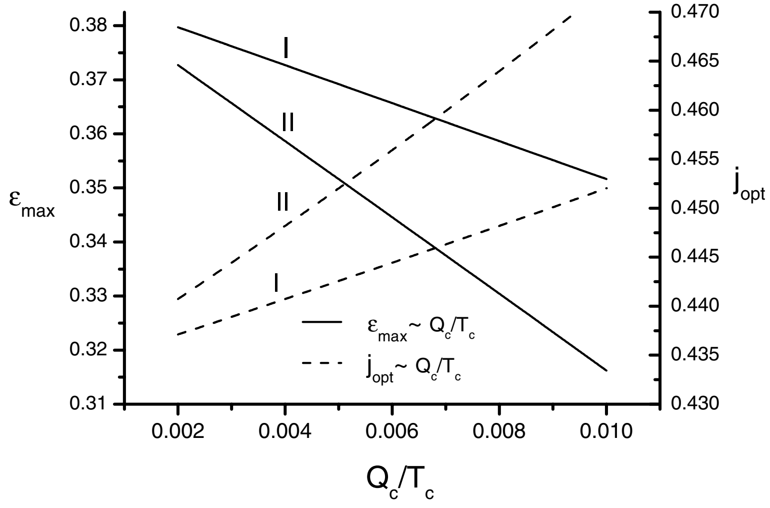

Fig. 3. The maximum coefficient of performance ε_{max} and the corresponding dimensionless current j_{opt} versus Q_c/T_c. The values of the parameters θ_h and ZT_h are the same as those used in Fig. 2 and curves I and II correspond to the cases of $U_cA = U_hA = 10\ \mathrm{W\ K^{-1}}$ and $5\ \mathrm{W\ K^{-1}}$, respectively.

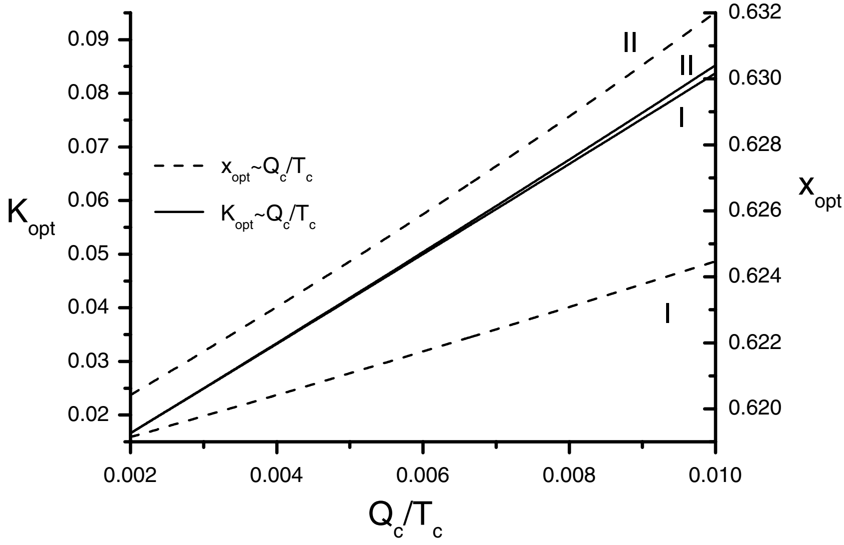

Fig. 4. The thermal conductance K_{opt} and area ratio x_{opt} at the maximum coefficient of performance versus Q_c/T_c. The values of the parameters θ_h, ZT_h, U_cA and U_hA are the same as those used in Fig. 3.

According to Eq. (14), the optimal characteristic curves obtained above, and the technological requirements, one can determine the optimal values of S_p/l_p and n. Consequently, Eq. (14) and the optimal characteristic curves of the system may provide theoretical guidance for engineers to design the structure of thermoelectric devices.

It is also seen from Fig. 4 that the optimal values of K, A_h/A and A_c/A are closely dependent on Q_c/T_c. This shows that for the thermoelectric refrigeration-system with

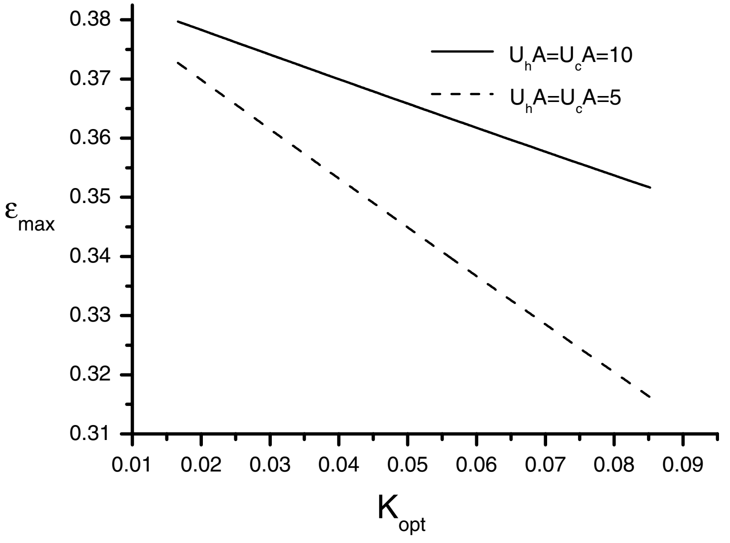

Fig. 5. The maximum coefficient of performance ε_{max} versus the thermal conductance K_{opt} at the maximum coefficient of performance. The values of the parameters θ_h, ZT_h, U_cA and U_hA are the same as those used in Fig. 3.

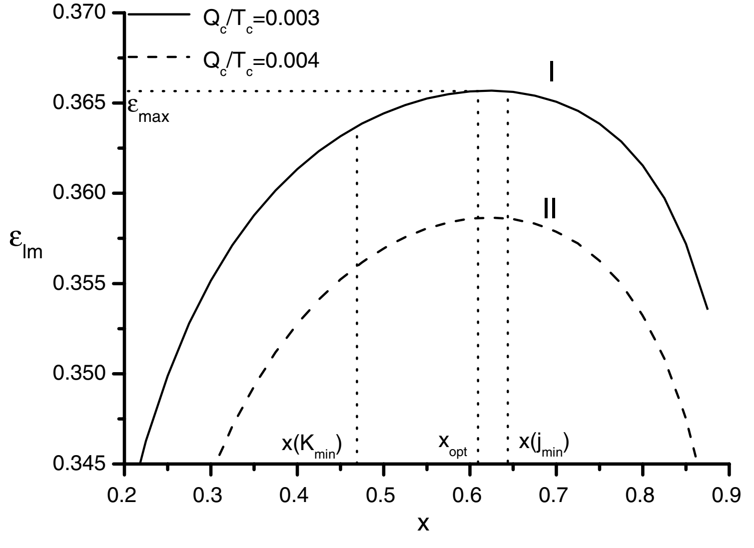

Fig. 6. The local maximum of the coefficient of performance ε_{lm} versus the area ratio x. The values of the parameters θ_h, ZT_h, U_cA and U_hA are the same as those used in Fig. 2 and curves I and II correspond to the cases of $Q_c/T_c = 0.003$ W K^{-1} and 0.004 W K^{-1}, respectively.

the different operative conditions which are required practically, the optimal values of the structure parameters $n(S_p/l_p)$ of the thermoelectric device and the area ratio of the heat exchangers must be different from each other. This is worthwhile to consider when designing the heat exchangers of the multi-couple thermoelectric refrigeration-system.

In addition, for given parameters x, Q_c/T_c, θ_h, ZT_h, U_cA and U_hA, using the extremal conditions $(\partial\varepsilon/\partial K)_x = 0$ and Eqs. (10) and (12), one can obtain the relations of the variation of the local maximum of the coefficient of performance and corresponding parameters K and j with the area ratio x, as shown in Figs. 6–8, respectively, where the parameters $\theta_h = 1.2$, $U_cA = U_hA = 5\ \mathrm{W\,K^{-1}}$ and $ZT_h = 1$ are chosen and curves I and II correspond to the cases of $Q_c/T_c = 0.003\ \mathrm{W\,K^{-1}}$ and $0.004\ \mathrm{W\,K^{-1}}$, respectively. It is seen from these figures that the local maximum of the coefficient of performance of the system ε_{lm} first increases and then decreases while the corresponding optimal values j_{lo} and K_{lo} of the dimensionless current and thermal conductance first decrease and then increase as x is increased. It shows that there is an optimal value of x at which ε_{lm} attains its maximum value, while j_{lo} and K_{lo} attain their minimum values for other given parameters. Also the optimal values of x are different for the different parameters at the different operating conditions. It can be seen from the curves in Figs. 6–8 that there is an important relation:

$$x(K_{\min}) < x_{\mathrm{opt}}(\varepsilon_{\max}) < x(j_{\min}) \tag{15}$$

where $x_{\mathrm{opt}}(\varepsilon_{\max})$ is the area ratio at the maximum coefficient of performance, and $x(K_{\min})$ and $x(j_{\min})$ are, respectively, the area ratios at the minimum thermal conductance and minimum dimensionless current. Figs. 9 and 10 further show the variations of ε_{lm} with K_{lo} and j_{lo}, respectively. It is seen from Figs. 9 and 10 that, when $j_{lo} > j_{\min}$ or $K_{lo} > K_{\min}$, there are two different coefficient of performance for a given dimensionless current j_{lo} or thermal conductivity K_{lo}. One always wants to obtain the coefficient of performance as large as possible for a given cooling load Q_c/T_c. When $x > x(K_{\min})$, the coefficient of performance of the system is larger than $\varepsilon(K_{\min})$. Similarly, when $x < x(j_{\min})$, the coefficient of performance of the system is also larger than $\varepsilon(j_{\min})$. Consequently, it is found that the range of the area ratio x should be

$$x(K_{\min}) < x < x(j_{\min}) \tag{16}$$

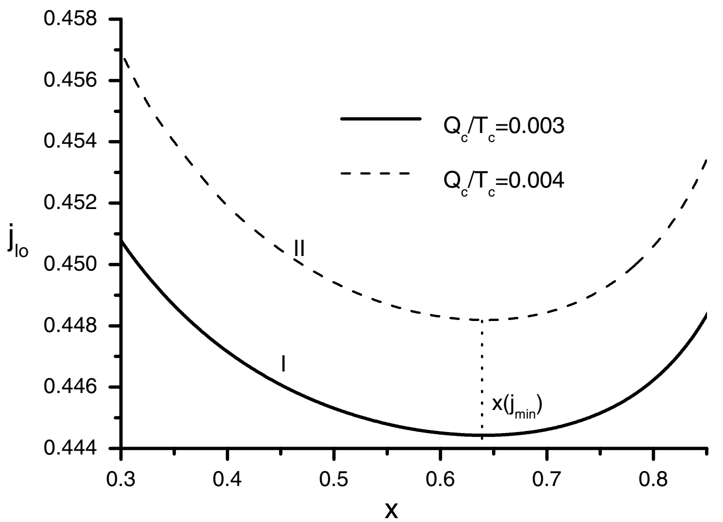

Fig. 7. The dimensionless current j_{lo} at the local maximum of the coefficient of performance versus the area ratio x. The values of the parameters θ_h, ZT_h, U_cA, U_hA and Q_c/T_c are the same as those used in Fig. 6.

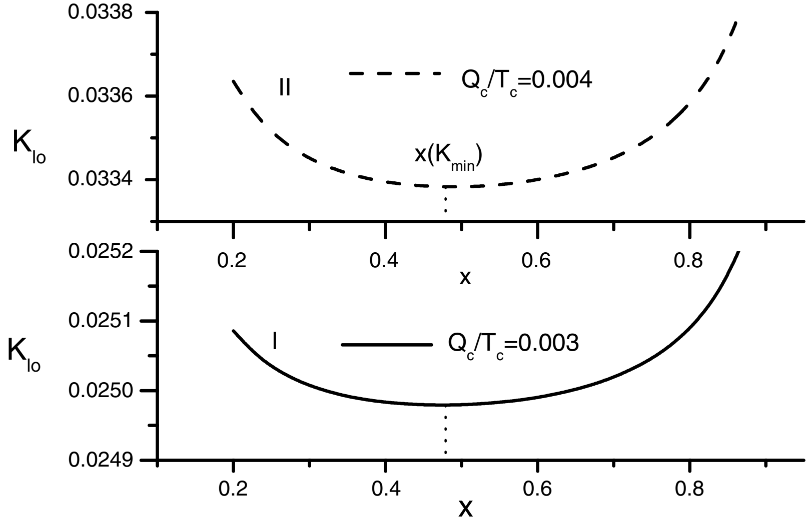

Fig. 8. The thermal conductance K_{lo} at the local maximum of the coefficient of performance versus the area ratio x. The values of the parameters θ_h, ZT_h, U_cA, U_hA and Q_c/T_c are the same as those used in Fig. 6.

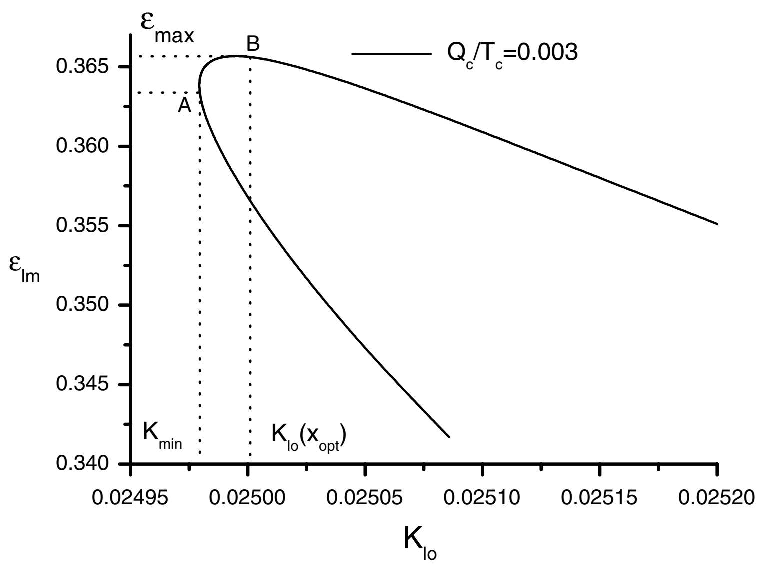

Fig. 9. The local maximum of the coefficient of performance ε_{lm} versus the thermal conductance K_{lo} at the local maximum of the coefficient of performance. The values of the parameters θ_h, ZT_h, U_cA, U_hA and Q_c/T_c are the same as those used in Fig. 6.

Only when $x = x_{opt}$, can the coefficient of performance of the system attain its maximum and the multi-couple thermoelectric refrigerator system be operated in the optimal state. In this case, the thermal conductance K, dimensionless current j and the structure param-

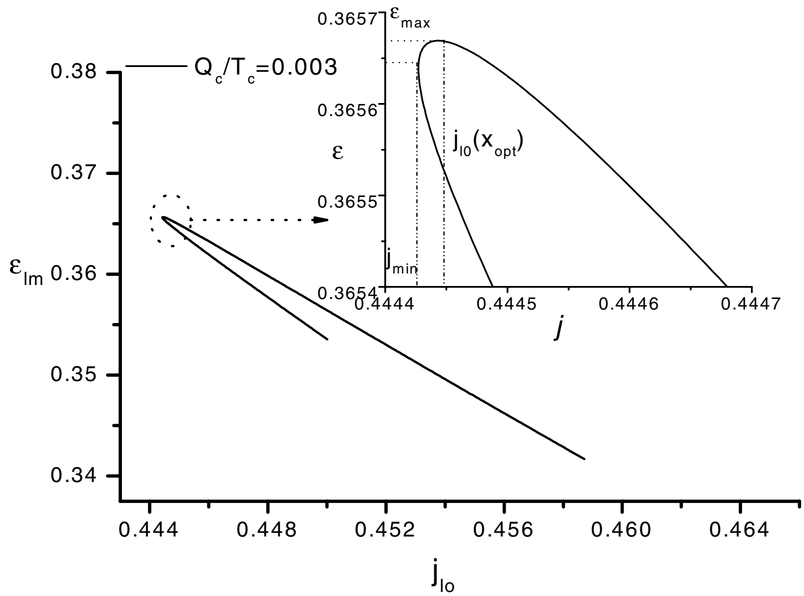

Fig. 10. The local maximum of the coefficient of performance ε_{lm} versus the dimensionless current j_{lo} at the local maximum of the coefficient of performance. The values of the parameters θ_h, ZT_h, U_cA, U_hA and Q_c/T_c are the same as those used in Fig. 6.

eters $n(S_p/l_p)$ of the thermoelectric device should be controlled to satisfy the following conditions:

$$K = K_{lo}(x_{opt}) \equiv K_{opt} \tag{17}$$

$$j = j_{lo}(x_{opt}) \equiv j_{opt} \tag{18}$$

and

$$n\frac{S_p}{l_p} = \frac{K_{opt}}{\kappa_p + \sqrt{\kappa_p\kappa_n\rho_n/\rho_p}} \equiv \left(n\frac{S_p}{l_p}\right)_{opt} \tag{19}$$

The above results show that ε_{max}, x_{opt}, j_{opt}, K_{opt} and $(nS_p/l_p)_{opt}$ are important parameters of a multi-couple thermoelectric refrigeration system, which are very useful for the optimal design of real thermoelectric refrigeration-systems.

4. Conclusions

The externally and internally irreversible multi-couple thermoelectric device model is used to analyze the performance of a thermoelectric refrigeration system. Some fundamental relations are derived, from which several optimal performance characteristic curves of the thermoelectric refrigeration system are plotted. The optimal operating-regions of some important performance parameters are specified. The optimal operating-state of the maximum coefficient of performance for a thermoelectric refrigeration device has been determined. The influence of the thermal conductance between the device and the heat

reservoirs and the cooling load on the performance of the system has been analyzed quantitatively. The results obtained here reveal some general performance characteristics of real multi-couple thermoelectric refrigeration systems operating at various conditions and could be used to guide the optimal design and manufacture of real thermoelectric-refrigerators.

Acknowledgements

This investigation has been supported by the Foundations of Science and Technology Projects of Fujian and Xiamen, People's Republic of China.

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